

Problem 1.41

[Difficulty: 2]

1.41 Repeat the calculation of uncertainty described in Problem 1.40 for air in a hot air balloon. Assume the measured barometer height is 759 mm of mercury with an uncertainty of ± 1 mm of mercury and the temperature is 60°C with an uncertainty of $\pm 1^\circ\text{C}$. [Note that 759 mm of mercury corresponds to 101 kPa (abs).]

Given: Air in hot air balloon

$$p = 759 \pm 1 \text{ mm Hg} \quad T = 60 \pm 1^\circ\text{C}$$

Find: (a) Air density using ideal gas equation of state
(b) Estimate of uncertainty in calculated value

Solution: We will apply uncertainty concepts.

Governing Equations: $\rho = \frac{p}{R \cdot T}$ (Ideal gas equation of state)

$$u_R = \pm \left[\left(\frac{x_1}{R} \frac{\partial R}{\partial x_1} u_{x_1} \right)^2 + \dots \right]^{\frac{1}{2}}$$
 (Propagation of Uncertainties)

We can express density as:

$$\rho = 101 \cdot 10^3 \times \frac{\text{N}}{\text{m}^2} \times \frac{\text{kg} \cdot \text{K}}{287 \cdot \text{N} \cdot \text{m}} \times \frac{1}{333 \cdot \text{K}} = 1.06 \frac{\text{kg}}{\text{m}^3} \quad \rho = 1.06 \frac{\text{kg}}{\text{m}^3}$$

So the uncertainty in the density is:

$$u_\rho = \pm \left[\left(\frac{p}{\rho} \frac{\partial \rho}{\partial p} u_p \right)^2 + \left(\frac{T}{\rho} \frac{\partial \rho}{\partial T} u_T \right)^2 \right]^{\frac{1}{2}}$$

Solving each term separately:

$$\frac{p}{\rho} \frac{\partial \rho}{\partial p} = RT \frac{1}{RT} = 1 \quad u_p = \frac{1}{759} = 0.1318\%$$

$$\frac{T}{\rho} \frac{\partial \rho}{\partial T} = \frac{T}{\rho} \left(\frac{-p}{RT^2} \right) = -\frac{p}{RT} = -1 \quad u_T = \frac{1}{333} = 0.3003\%$$

Therefore:

$$u_\rho = \pm \left[(u_p)^2 + (-u_T)^2 \right]^{\frac{1}{2}} = \pm \left[(0.1318\%)^2 + (-0.3003\%)^2 \right]^{\frac{1}{2}}$$

$$u_\rho = \pm 0.328\% \left(\pm 3.47 \times 10^{-3} \frac{\text{kg}}{\text{m}^3} \right)$$